JURNAL EDUSCIENCE (JES)
p-ISSN : 2303 - 355X | e-ISSN : 2685 - 2217
PUBLISHED BY : LPPM of UNIVERSITAS LABUHANBATU

The I-CON Model in Constructing Mathematical Proof

Abdul Mujib[1*], Firmansyah[2], Siti Shara Siagian[3], Devi Mathelinea[4]

[1,2,3]Mathematics Education, Universitas Muslim Nusantara Al-Washliyah, Indonesia
[4]Mathematics, Universiti Tun Hussein Onn Malaysia, Parit Raja Malaysia
*Email: mujib@umnaw.ac.id

ARTICLE INFO

Keywords:
Learning Model
ICON-Model
Mathematical Proof

ABSTRACT

Purpose-This research aims to analyze the role of the I-CON model in constructing mathematical proofs.

Methodology-The research used is qualitative with a grounded theory approach. Respondents were selected using a theoretical sampling approach, based explicitly on concepts that have been shown to relate to the theory being developed. Analysis data is obtained based on student test results, which are given to respondents, compiled into a new concept or theme, and then the desired subcategory.

Findings- The theory derived from this research is that, through the I-CON model, students can construct robust, precise, and valid mathematical proofs. The implementation of the I-CON model in the ability to construct mathematical proofs is (1) students can link facts with properties to interpret existing problems, (2) students can sequence valid proof steps, (3) students can use premises, definitions, and theorems related to statements to build a proof, (4) students can use appropriate arguments in the proof process, (5) Students have a systematic flow of thinking so that the proof steps are consistent, and (6) Students can interpret symbols mathematical and use precise mathematical communication language, which is obtained through learning the ICON model. Through learning the I-CON model, students can have the ability to understand various concepts, theorems, and definitions. They can make conjectures from statements given by interpreting them in detail. Implementing the Interpretation-Construction Design (I-CON) model in constructing mathematical proof produces six categories: Initial steps of proof, Flow of Proof, Related concepts, Arguments, Interpretation, and Language of Proof.

Significance-The results emphasize the importance of students constructing interpretations of real-world problem situations, discussion activities in building interpretations, reflecting, analyzing, and concluding interpretations that students construct as the primary focus of learning activities.

Received 20 February 2025; Received in revised form 28 February 2025; Accepted 14 June 2025
Jurnal Eduscience (JES) Volume 12 No. 3 (2025)
Available online xx June 2025
©2025 The Author(s). Published by LPPM Universitas Labuhanbatu. This is an open-access article under the **Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International License (CC BY - NC - SA 4.0)**

INTRODUCTION

Mathematics is fundamental in improving a person's intellectual abilities and skills (Siagian et al., 2022; Methkal, 2022; Manurung & Pappachan, 2025). Mathematics will shape a person to have the ability to think logically, systematically, analytically, creatively, and critically, as well as the ability to work together. Mathematics studies order, organized structures, and mathematical concepts are arranged hierarchically, structured and systematically, from the simplest to the most complex. Learning mathematics makes it a habit to solve problems by asking the right questions, looking at the available facts, distinguishing them from assumptions, and solving the problem with creative and systematic solutions (Sianturi, 2021; Haryadi et al., 2024; Prabandari et al., 2024). Meanwhile, the fundamental concepts of mathematics that need to be understood are the concept of reflection and definition of concepts as well as the ability to abstract from the material to be studied which is obtained from previous knowledge (Mutianingsih et al., 2025; Kurniawan et al., 2024; Ye et al., 2023). Abstraction abilities will be described through problems that become mathematical concepts in constructing proof by building problem situation models according to related concepts. So, students will learn the nature of general concepts/theorems related to the material and abstract concepts from the material (Suwanto et al., 2017).

Mathematical modelling ability is one of the many mathematical abilities students must have. Several previous studies have concluded that students are less skilled in finding important elements contained in the problem, and reflection of mathematical models and solutions produced; limited student knowledge in solving real-world problems (Zulkarnaen, 2018a). Students' answers are still wrong when doing informal mathematization, and the answers given are irrelevant to the problem formulation and the problem-solving procedure, and do not match the solution with the problem situation presented—the relationship between real-world problems and mathematics through mathematical models. Mathematical modelling helps students to understand and use mathematics in the real world and see the connection between mathematics and the real world. Therefore, learning mathematics with the nature of 'transferring' learning materials directly hurts students in creating and explaining a mathematical model related to the problem presented. Giving questions in the context and situation of the real world can improve students' mathematical modelling abilities (Zulkarnaen, 2020).

The ability to construct students' mathematical proofs through the I-CON Model will be seen in abilities that include the ability to draw, mathematical expressions, and written text. This is by the aspects used to measure the ability to construct mathematical proof in this research, namely: 1) expressing a mathematical situation or idea in the form of an image and completing it (drawing), 2) expressing a mathematical situation or idea in the form of a symbol or model mathematics and solving it (mathematical expression/mathematical model), and 3) stating and explaining an image or mathematical model in the form of mathematical ideas (written texts) (Suwanto et al., 2017). By applying the I-CON Model, students will have superior abilities in constructing mathematical evidence because they are more motivated and organized in learning. Students can express a mathematical situation or idea as an image in problem-solving (Nugraha & Pujiastuti, 2019).

The interpretation-construction design (I-CON) model is implemented to develop the ability to construct mathematical proof, which contains the principles of observation in authentic activities, interpretation construction, contextualization, cognitive apprenticeship, collaboration, multiple interpretation, and multiple manifestation (Black & McClintock, 1996; Tsai, 2001). Then the I-CON principle of the model will be linked to aspects of constructing mathematical proof. Mathematics learning using the Interpretation-Construction Design model (I-CON model) places more emphasis on the importance of students constructing interpretations from real-world problem situations, discussion activities in building interpretations, reflecting, analyzing, and concluding interpretations that students construct as the primary focus of learning activities, teachers act as facilitators in providing a learning environment, and students are actively involved in building students' knowledge independently (Zulkarnaen, 2018b). The I-CON model is very suitable for the learning material that will be used, namely transformation geometry. To solve it, we must first investigate the properties, which we will then interpret and construct into a mathematical proof.

However, according to field facts, constructing proof through interpretation is still a problem for several students (Selden, 2003; Stylianou et al., 2015; Stylianides et al., 2024). This is indicated by the statement that students experience difficulty in abstract thinking when completing the proof construction process with valid and relevant steps. Considering the importance of students' abstraction abilities in solving real problems, it is necessary to discuss the ability to construct mathematical proofs in the application of the I-CON model of learning. Knowledge about this can be considered for researchers designing an alternative to improve the ability to construct mathematical proof using the I-CON model. Considering the importance of learning the I-CON model in constructing mathematical proofs, this is a reason for researchers to carry out this research, namely, how to implement the I-CON model in constructing mathematical proofs.

METHODOLOGY

Research Design

The research method used in this research is descriptive qualitative research with a Grounded Theory approach (Susanto et al., 2024). The respondents selected were 10 students. Participants were selected using theoretical sampling according to the concepts needed in implementing the I-CON model in constructing mathematical proof. The selection of research locations is based on the curriculum structure of the research location. In addition, the selection of the research location was not carried out at the author's home institution for the reason that there was a possibility that research respondents would have a negative perception of the assessment process. The I-CON model learning syntax and mathematical modelling component aspects can be seen as shown in the following picture:

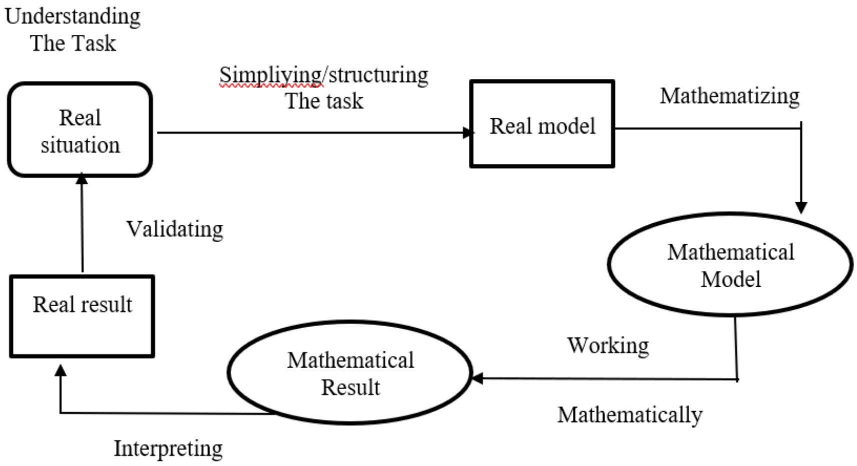


Figure 1. ICON Model learning steps

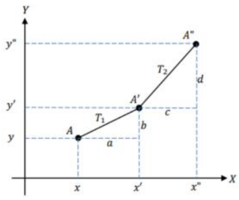
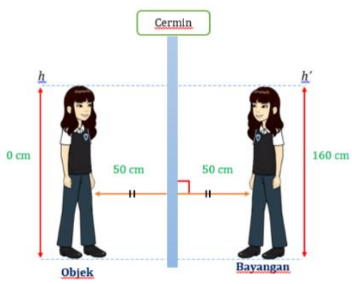
The steps in this research include: 1) giving students a test of their ability to construct mathematical proof through the I-CON Model in the form of 3 essay questions; 2) carry out an analysis of the results of students' work on the ability test to construct mathematical proof after learning the I-CON model; 3) conducting interviews based on the results of students' work, 4) analyzing the results of students' work on the ability test to construct mathematical proof through I-CON learning and the results of interviews with students. This grounded theory research consists of three sequential steps, namely open coding, selective coding, and theoretical coding (Jones & Alony, 2011).

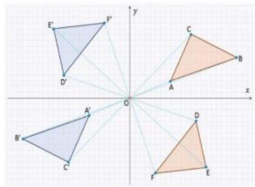
Data obtained from the ability test to construct mathematical proof will be analyzed descriptively to find out the impact provided by learning the I-CON model. Next, the data obtained from the test will be analyzed in order to find out how to implement the I-CON learning model given to participants. Analysis of student errors was carried out to determine the level of student difficulty in solving problems on the ability to construct mathematical proofs in transformation geometry material with types (Kingsdorf & Krawec, 2014), interviews were conducted to determine students' perceptions of learning mathematics using the I-CON model, and 'confirmation' of student error.

FINDINGS

After giving the following transformation geometry questions:

Table 1. Mathematical Proof Construction Questions through I-CON Model learning

No.	Material	Question Script
	Translation	The form of translational composition can be observed in the figure below. Point A is translated by $T_1 = \begin{pmatrix} a \\ b \end{pmatrix}$ to produce point A', then point A' is translated again by $T_2 = \begin{pmatrix} c \\ d \end{pmatrix}$to produce point A." Such a process is called Translational Composition.  From the picture above it can be stated that: The translation composition at point A can be written as: $(x, y) \xrightarrow{\begin{pmatrix} a \\ b \end{pmatrix}} (x + \cdots, y + \cdots) \xrightarrow{(\cdots)} (x + \cdots + \cdots, y + \cdots + \cdots)$ Expressed in matrix form: $A' \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cdots \; \cdots \\ \cdots \; \cdots \end{pmatrix} + \begin{pmatrix} \cdots \; \cdots \\ \cdots \; \cdots \end{pmatrix} + \begin{pmatrix} \cdots \; \cdots \\ \cdots \; \cdots \end{pmatrix} = \begin{pmatrix} + \quad + \\ + \quad + \end{pmatrix}$ Draw a conclusion from the statement above regarding the composition of the shift.
2	Reflection	Rani stands in front of the mirror at a distance of 50 cm and Rani's height is 160 cm. What is Rani's reflection in the mirror? How far is Rani's image from the mirror? Look at the following image illustration:  Find the concept from the image pattern above!
3	Reflection	My children, to understand the concept of reflection at the origin O(0, 0), let's observe the reflection of triangle ABC and triangle DEF. How do each point A, B,

		C in triangle ABC and points D, E, F in triangle DEF change after being reflected at the origin, namely point O(0, 0)?  In the picture above, we can see that triangle A'B'C' is a shadow of triangle ABC after being reflected at the origin O (0,0). Triangle D'E'F' is the image of triangle DEF after being reflected at the origin O (0,0). Children can easily understand the changes in coordinates of each point that occur in triangle ABC and triangle DEF. Make a table to state the points from the image above, then draw a conclusion from the table that has been made regarding the reflection formula. After that, make proof and conclusions from the reflections obtained in matrix form.

The results of the test for the ability to construct mathematical proof through the application of the I-CON Model learning are as follows:

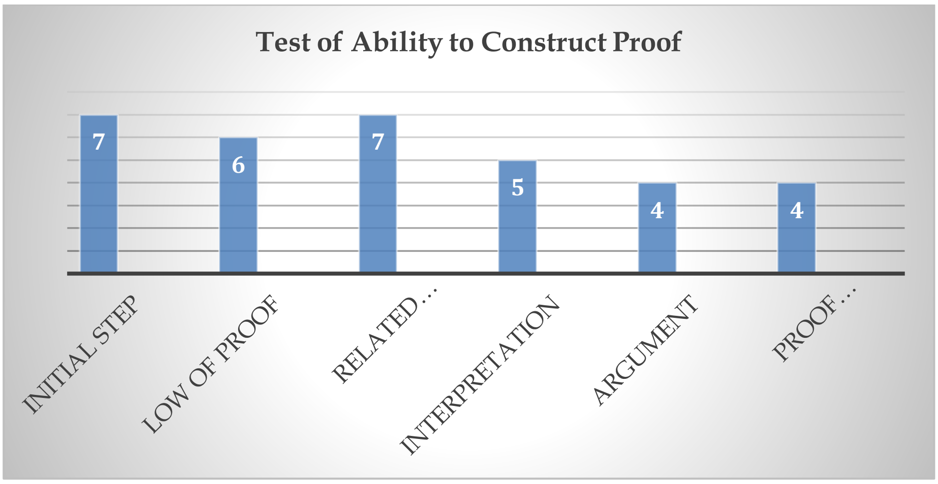


Figure 2. Graph of Test Results for the Ability to Construct Mathematical Proof

Based on the results of the graphic above, it shows that after linking aspects of constructing mathematical proof with I-CON learning, the model produces six categories, namely initial steps, proof flow, related concepts, interpretation, arguments and language of proof. Referring to the graph, it can be seen that from the 10 participants it was found that: 1) there were 7 students who were able to choose the right initial step in solving the problem given, 2) there were 6 students who used the correct and valid proof flow because they had implemented I-CON learning model, 3) there are 7 students who have been able to master and understand the use of related concepts in constructing mathematical proofs through the application of I-CON model learning, 4) there are 5 students who have been able to interpret aspects of constructing mathematical proofs, 5) there are 4 students who are able to use argue appropriately and logically in constructing mathematical proof through learning the I-CON model, and 6) there are 4 students who are able to use appropriate proof language during the process of solving the questions given.

To understand in more detail, the results of the answers to the test for the ability to construct mathematical proofs through learning the I-CON Model will be described as follows:

Initial Step

The first category studied is regarding the diversity of determining the initial steps in constructing mathematical proof. Figure 3 is an example of determining a student's initial steps correctly.

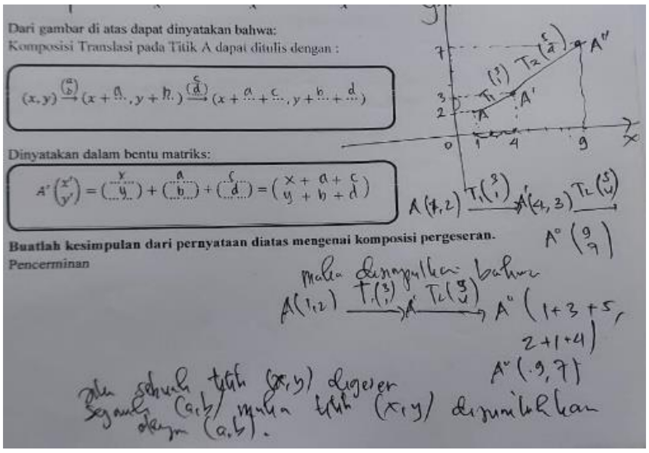


Figure 3. Determination of Initial Steps for question number 1 by R-5

Based on the figure above, we can see that R-5 has the ability to identify assumptions related to material concepts and things that are known in the statement to be proven, and students are able to utilize assumptions about concepts appropriately as capital in determining appropriate proof construction steps. with the I-CON learning principle, the first model is making authentic observations which are then presented using communicative mathematical language. Figure 4 is a determination of the student's incorrect initial steps.

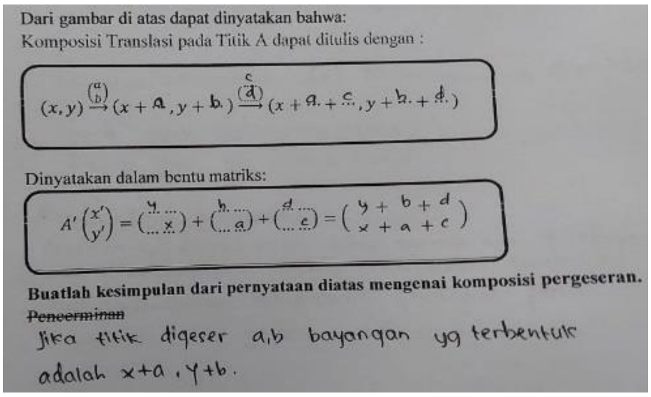


Figure 4. Determination of Initial Steps for question number 1 by R-1

There are differences in answers between R-5 and R-1, if seen from the results of their textual work. R-5 with high ability is able to explain every assumption and thing that is known into a more significant and specific form in the first step of proof. The initial steps made by R-5 have illustrated that high ability students have led to the required proof process. In some cases, the initial steps expressed by R-5 show that he already knows the final goal of the requested proof, because the learning principles of the I-CON model have been implemented well. while answer R-1 shows that it is unable to determine the correct initial step so the answer presented is wrong. The delivery of the initial steps written by R-5 rather than R-1 is more logical because the language used is easier to understand than R-1.

Flow of Proof

The second category studied in this research is the diversity of accuracy of writing the flow of proof in constructing proof through learning the I-CON Model. Figure 5 below is an example of a respondent's work that uses the correct proof flow.

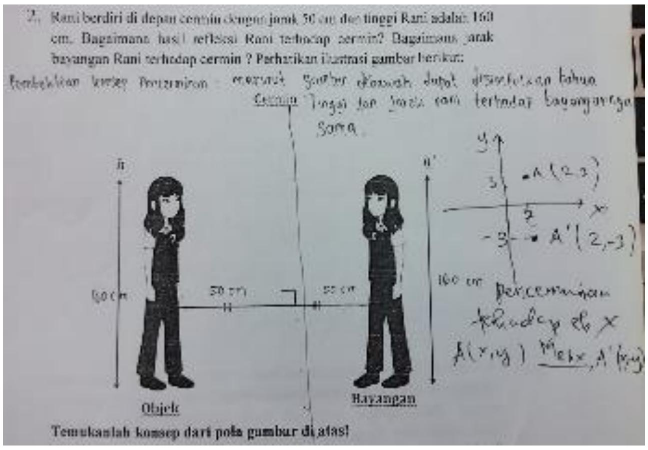


Figure 5. Flow of Proof of question number 2 by R-8

Based on the figure above, it can be seen that R-8 has a level of proficiency in using clear and precise proof lines or strategies. The use of the I-CON Model principle in constructing mathematical proofs is used to accurately reflect the respondent's train of thought in a coherent manner, clearly in accordance with the flow of proof that should be used. The steps taken by R-8 reflect a coherent line of thinking and do not contain leaps of logic. To see the diversity of answers produced, it can be seen in Figure 6 below.

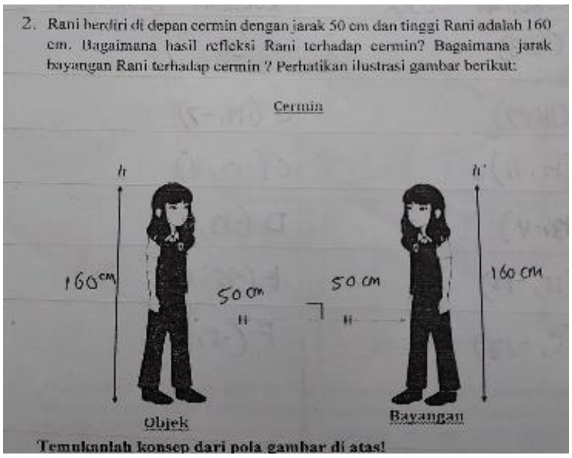


Figure 6. Flow of Proof of question number 2 by R-1

The figure above shows that the line of thinking shown by R-1 was an error, including: 1) the line of thought in the overall proof compiled was not clear. The meaning of the image pattern of the questions is not correct, 2) there is a lack of understanding and doubts by students in interpreting the questions given, 3) using erroneous statements which are considered to fulfill what is required to make conclusions, 4) unclear strategies used, 4) unable to direct their work on the construction of a complete proof. So, R-8's answer is better than R-1, because the sequence of solutions carried out by R-8 is more regular.

Related Concepts

The third category studied is the understanding and use of concepts related to the construction of evidence through I-CON model learning which greatly determines the quality of the proof process and the use of mathematical language that will be compiled and used in representing the implementation of I-CON model learning. Seen in the following figure.

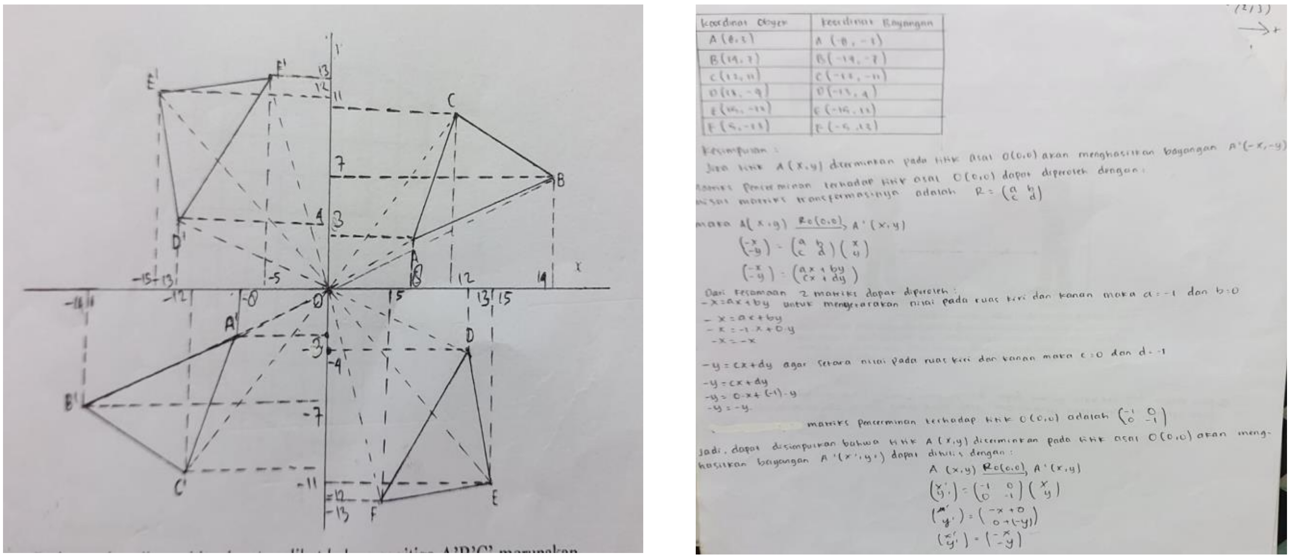


Figure 7. Concepts Related to question number 3 by R-5

Based on the figure above, we can see that answer R-5 has a good level of understanding and utilization of concepts related to the category. Understanding and utilizing related concepts in the construction of evidence through learning the I-CON model greatly determines the quality of the proof process and the use of mathematical language that will be compiled and used in representing answers. The concept related to question number 3 is the use of the properties and characteristics of reflection in determining the object/image point, as well as the final conclusion from the reflection concept obtained. The following image will show an example of using concepts with inappropriate categories.

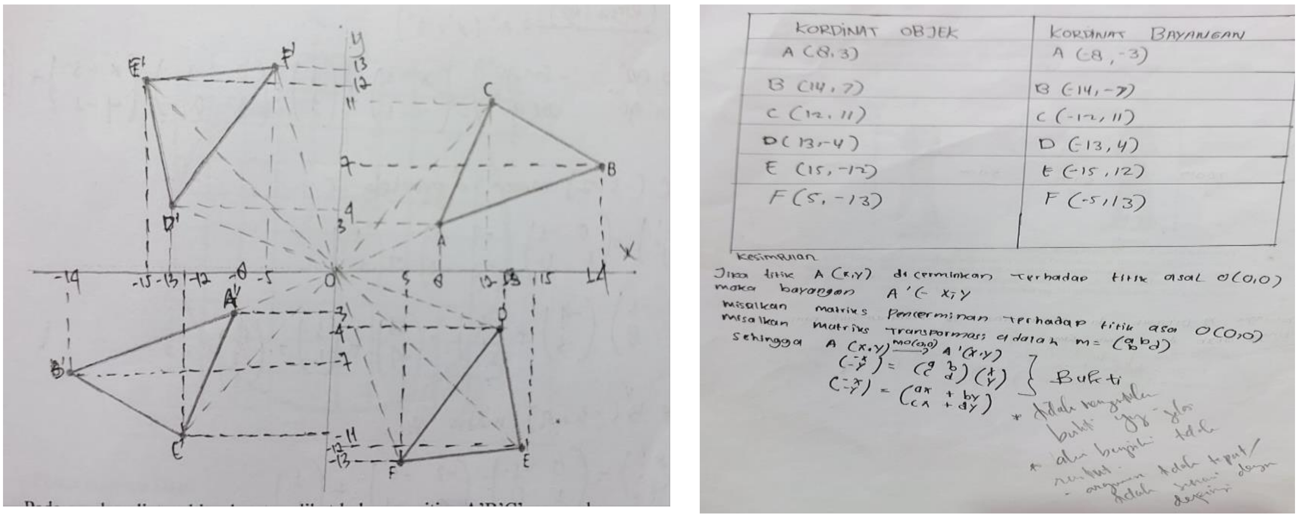


Figure 8. Concepts Related to question number 3 by R-2

The figure above shows that answer R-2 does not have accuracy in understanding and utilizing related concepts. Errors and difficulties in using and mastering and utilizing the I-CON model of learning in constructing proof experienced by students include: 1) errors in explaining the meaning of the image in the question, namely R-2, wrong in determining the elements of proof that must be used in proving the formula. has been acquired and the language used is not communicative, 2) the concepts needed to determine the initial steps are not mastered, 3) weak understanding of the concepts required in part or the entire construction of proof using the principles of the I-CON model, 4) errors in proof and interpretation strategies I-CON model principles in constructing, so that known concepts are not utilized appropriately, 5) students do not get a definite picture from the I-CON model principles regarding reflection (mirroring) material towards the origin

O (0,0), 6) students do not master and understand the definition of the problem given, and 7) students do not master the concepts needed to determine the initial steps in the process of constructing mathematical proof and communication.

Argument

The fourth category in this research study is the preparation of appropriate arguments. Figure 9 will show the correct arrangement of the following arguments.

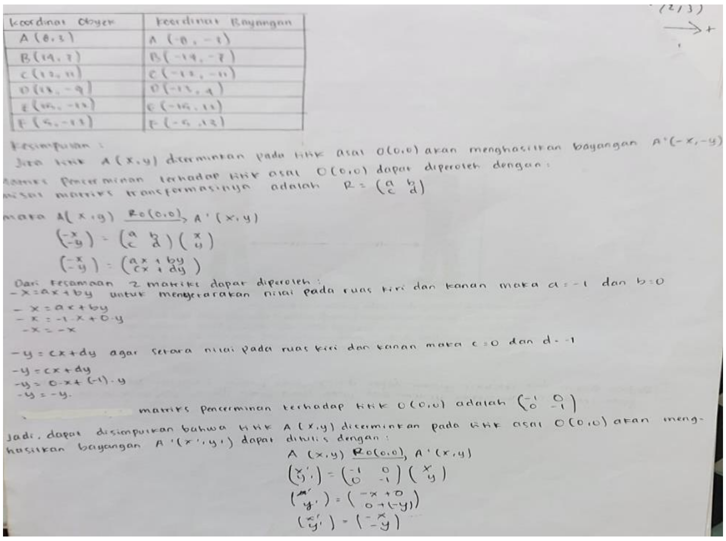


Figure 9. Preparation of Arguments for question number 3 by R-8

Based on the figure above, it shows that R-8 has precision and accuracy in preparing arguments. The precision and accuracy of an argument is intended as the basis for a message that will be conveyed/interpreted, either orally, in writing, in performance or in expressions intended to support a step or a statement that will be expressed in the form of a presentation using the I-CON model learning principles. The precision and accuracy of the argument greatly determines the quality of the construction of proof. The following figure shows an incorrect argument arrangement.

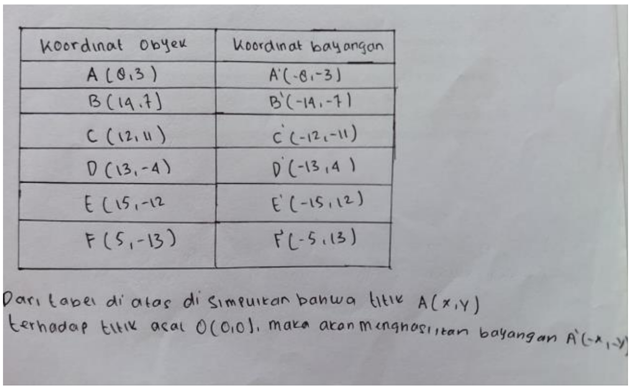


Figure 10. Arrangement of Arguments for question number 3 by R-2

Based on the figure above, it shows that R-2 is unable to organize arguments accurately and logically without being accompanied by logical reasons. The inaccuracies in the arguments found were: 1) weak mastery of related concepts, making the arguments given inappropriate. For example, the conclusion about the validity of the theorem used with the arguments presented is incorrect due to an error in understanding the definition of the image. 2) not writing arguments in parts where arguments should be required. For example, writing A' (x', y') becomes A'(-x,-y) without supporting arguments, 3) errors in conveying arguments due to not being careful in distinguishing the correct use of statements, 4) arguments put forward are not sufficient supporting the truth of the statement from the image provided, 5) confusion in students' understanding of the problem in the form of an image as well as the use of theorems and the concept of reflection (mirroring) towards the origin point O (0,0) which applies to the problem presented, 6) arguing with the nature/statement However, the characteristics/statements are not appropriate to support the arguments put forward or are not in accordance with the facts contained in the question, 7) students' lack of understanding and mastery of the concept of image in interpreting the images presented so that they do not understand the use of arguments that should be written in constructing proof and mathematical communication of the problem, so that the arguments put forward and interpreted are weak or even inaccurate. For example, using arguments from the nature of the definition of reflection (reflection) towards the origin point O (0,0) to provide logical reasons for the process of constructing proof and using mathematical language, 8) using arguments that are not basic or have no connection to the problem, for example final conclusions from the concepts and types of reflection (reflection) contained in the images presented therein do not show what they should, only based on arguments according to their respective views without linking them to the theorems or concepts that should be used.

Interpretation

The fifth category of this research study is interpreting answers clearly and validly in accordance with the aspects and principles of constructing mathematical proof that comes from implementing the I-CON model. The following is a figure of the respondents' answers regarding the interpretation of the answers.

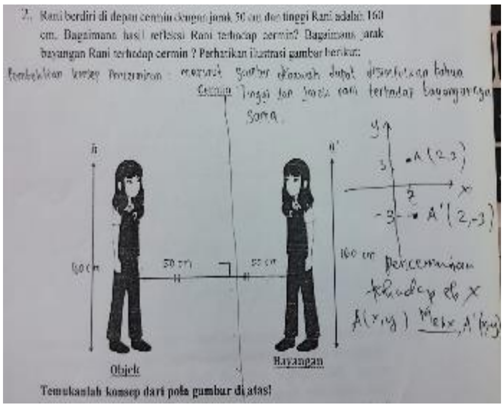


Figure 11. Interpretation of question number 2 by R-8

Based on the figure above, it shows that R-8's answer has been interpreted correctly by using all the principles in learning the I-CON model to construct mathematical proof. The more complete the proof steps that are constructed, the better the results of the level of understanding regarding the application of the interpreted model of I-CON learning, which shows that the mastery of understanding the concept is also good. The following is an interpretive figure presented by R-9.

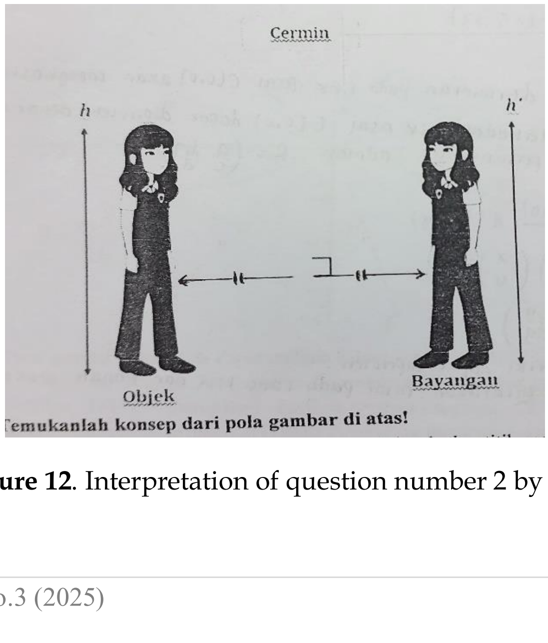


Figure 12. Interpretation of question number 2 by R-9

Referring to the figure above, it shows that there were several mistakes made by R-9 in interpreting the questions into answers, including: 1) not being able to interpret the image patterns presented so they did not know how to interpret the answers, what should be used, 2) no understand and pay attention to image patterns carefully, and 3) students are not able to translate the questions well. It was found that R-8 better understood and could utilize various assumptions that should be used in finding formulas from the image patterns presented, and in question number 2, R-8 was superior in using verbal language in interpreting the findings he obtained. Meanwhile, R-9 was not able to make good use of the assumptions they had so that they were unable to interpret question number 2 correctly as the answer should be.

Proof Language

The sixth category of this research study is language of proof. In constructing mathematical proof, the language of proof is the key to solving problems, especially with the application of the I-CON model. The figure is an example of the use of proof language.

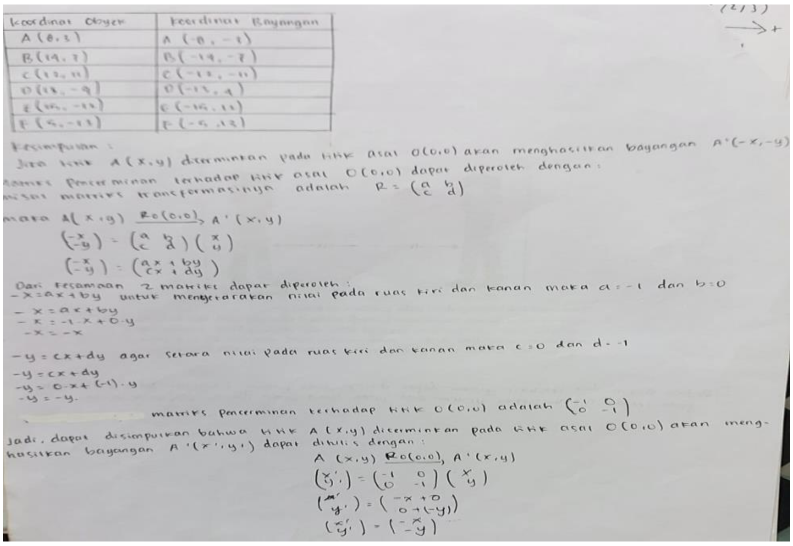


Figure 13. Language Proof of question number 3 by R-5

Based on the figure above, it shows that R-5 good proof construction must use proof language and mathematical language that is communicative and meaningful within the reach of the class community (the class concerned). The language of proof written by R – 5 students is very communicative and does not have the potential to cause confusion or ambiguity for the reader. Starting with determining the nature and characteristics of the concept of reflection (reflection) which is accompanied by a detailed explanation, then concluded by writing the concept of reflection (reflection) in full, including all the elements related to it. The image pattern is converted in graphic form into the coordinate points of the original object and the resulting image is written down and utilized properly as an explanation of the axioms or characteristics/properties of reflection (reflection) which are shown to be true.

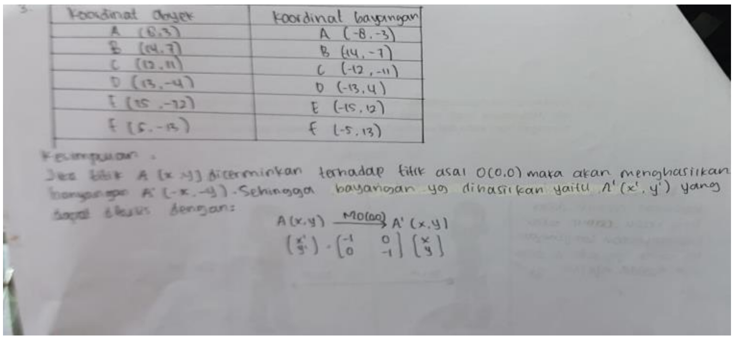


Figure 14. Use of Language Proof of question number 3 by R-1

Based on the picture above, we can see that R-1 in choosing words is not correct so that the statement sentences used contain leaps of logic, this is also a student error in the process of constructing proof. In question number 3, R-1 made more mistakes in using uncommunicative language in the proof process. The language compiled by R-1 does not describe and does not show a communicative proof structure. There is no complete explanation of the nature and characteristics of translation concepts or theorems. Likewise, regarding the exposure of object points and shadows, there is no clear explanation as to why this conclusion can be obtained so that it does not contain the learning principles of the I-CON model.

DISCUSSION

The application of the ICON model has a significant impact on students in solving mathematical proof construction problems. This aligns with Bond (2020) and Alon et al (2019). Starting with presenting real-world problems about transformation geometry, students try to understand and visualize concepts from the real world as mathematical concepts. The next step is for students to carry out the process of mathematization in constructing proofs by generating mathematical models. Constructing mathematical proofs produced by students is interpreted and validated through real-world conditions. The challenges faced by teachers in implementing the ICON-Model in mathematics education include the teacher's role in guiding students in constructing proofs, the strategies used to overcome students' learning obstacles, and the positive impact of using the ICON model on students' abilities, especially in mathematical proof construction.

Strategies to overcome the challenges in implementing the ICON-Model in mathematics learning can involve periodic teacher training and mentoring, collaboration among teachers to share experiences and practical strategies, and support from the school and government in providing the necessary resources. In addition, regular evaluations of the implementation of the ICON-Model are also necessary to assess its success and identify areas that still need improvement. With these efforts, implementing the ICON-Model in mathematics education can significantly positively impact the advancement of mathematics education in Indonesia.

The research results obtained six indicators in constructing proofs: initial steps, proof flow, related concepts, arguments, interpretation, and proof language. The initial step in constructing a mathematical proof is crucial, as it determines the subsequent steps (Harel & Larry, 2007). The findings show that most students fail to determine the initial step, which impacts their ability to construct proof. The proof path is related to the students' logical reasoning (Ball & Bass, 2003). The accuracy of the proof outline indicates the appropriateness of the strategies used by students in constructing the proof. The related concept is associated with students' understanding of mathematical concepts that support the construction of mathematical proofs. Arguments in proof construction play a role in determining the quality of mathematical proof construction (Hanna, 2020; Lin et al., 2004)—interpretation of proof construction as a form of mathematical communication language linked to the real world. Moreover, the language of proof uses logical and communicative mathematical language, so it does not have the potential to cause multiple interpretations for the reader (Rohid et al., 2019).

Mathematical modelling is a complex process compared to arithmetic skills. It does not rely solely on conception, as argued by (Zulkarnaen, 2018) (Zulkarnaen, 2020) students must understand (1) what mathematical structures are available; (2) aspects and elements that are relevant to the characteristics of the problem situation being modelled; and (3) how to justify the use of specific mathematical structures to represent aspects or elements identified from real-world situations (Zulkarnaen, 2018). Therefore, mathematical concept schemes are not enough to make students skilled in mathematization, and students do not know enough mathematical concepts and procedures (e.g., systems of linear equations and statistics). Mathematical modelling requires selecting and using appropriate mathematical concepts or procedures in representing real-world problems in mathematical form or constructing mathematical models (Slamet Kusumawardana & Diantarini, 2021) (Indriawati et al., 2017). Thus, students should not be accustomed to memorizing mathematical facts, rules, and procedures. However, students should also be able to explain how or why mathematical concepts and procedures are used in solving problems. In addition, the use of real-world problem situation contexts should be familiar, which can make students imagine themselves in the problem situation presented.

The interpretation-construction design model is implemented to develop mathematical modelling skills, including the principles of observations in authentic activities, interpretation construction, contextualization, cognitive apprenticeship, collaboration, multiple interpretations, and multiple manifestations (Indriawati et al., 2017). Mathematics learning using the interpretation-construction design model (hereinafter abbreviated as ICON-model) emphasizes the importance of students constructing interpretations from real-world problem situations, discussion activities in building interpretations, reflecting, analyzing, and concluding interpretations that students build as the primary focus of learning activities, teachers act as facilitators in providing a learning environment, and students are actively involved in building student knowledge independently (Indriawati et al., 2017; Kusumawati et al., 2024).

CONCLUSION

Based on the research results presented, it was found that learning mathematics using the Interpretation-construction design model significantly impacted students' ability to construct mathematical proofs. Applying the I-CON model learning principles with aspects of constructing mathematical proof are interconnected, so students' abilities in constructing proof are better than before. The categories resulting from the principles of learning the I-CON model by constructing mathematical proofs are 1) Initial steps of proof, 2) Flow of Proof, 3) Related concepts, 4) Arguments, 5) Interpretation, and 6) Language of Proof. However, students are still weak in providing interpretations related to mathematical models and verifying abstract mathematical models resulting from the questions given.

From the results of students' answers, we see that students in the high ability category will be able to interpret the answers well. While students with low ability categories will experience several errors in interpreting answers, including: 1) finding it difficult to translate questions, 2) technical errors occurring in interpreting images and using the correct symbols, and 3) students having difficulty determining the appropriate initial steps. Related to the correct conjecture, 4) there are incoherent steps, 5) the argument is illogical, 6) the flow of proof is incoherent, 7) there is no consistency in the proof steps, and 8) the interpretation of the answers presented is not systematic

ACKNOWLEDGMENT

This work was supported by the Rector of UMN Al-Washliyah for providing funding for the internal research grant.

REFERENCES

Alon, U., Zilberstein, M., Levy, O., & Yahav, E. (2019). code2vec : Learning Distributed Representations of Code. *Proceedings of the ACM on Programming Languages*, *3*(January), ISSN 2475-1421. https://doi.org/https://doi.org/10.1145/3290353

Ball, D. L., & Bass, H. (2003). Making Mathematics Reasonable in School. In *A Research Companion to Principles and Standards for School Mathematics* (Issue January 2003, pp. 27–44).

Black, J. B., & Mcclintock, R. O. (1996). An Interpretation Construction Approach to Constructivist Design Interpretation Construction (ICON) Design Model. In *Constructivist Learning Environments: Case Studies in Instructional Design* (Issue October, pp. 1–6).

Bond, M. (2020). Facilitating student engagement through the flipped learning approach in K-12 : A systematic review. *Computers & Education*, *151*(10), 103–819. https://doi.org/10.1016/j.compedu.2020.103819

Hanna, G. (2020). Mathematical Proof, Argumentation, and Reasoning. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education*. Springer. https://doi.org/10.4324/9780203825495

Harel, G., & Larry, S. (2007). Toward Comprehensive Perspectives on the Learning and Teaching of Proof. In *Second handbook of research on mathematics teaching and learning: a project of the National Council of Teachers of*

Mathematics (Vol. 6179, Issue 619, pp. 1–60).

Haryadi, J., Mujib, A., & Siagian, S. S. (2024). *Analysis of Student Image Concepts in Constructing Proof and Mathematical Communication in terms of Gender*. *6*(4), 103–109.

Indriawati, S., Sunardi, & Kurniati, D. (2017). Pengembangan Indikator 4 C's Yang Selaras Dengan Kurikulum 2013 pada Mata Pelajaran Matematika SMP/MTs Kelas IX Semester 1. *Kadikma*, *8*(2), 176–188.

Jones, M., & Alony, I. (2011). Guiding the use of grounded theory in doctoral studies - An example from the australian film industry. *International Journal of Doctoral Studies*, *6*(January), 95–114. https://doi.org/10.28945/1429

Kingsdorf, S., & Krawec, J. (2014). Error analysis of mathematical word problem solving across students with and without learning disabilities. *Learning Disabilities Research and Practice*, *29*(2), 66–74. https://doi.org/10.1111/ldrp.12029

Kurniawan, A., Jumadi, Kuswanto, H., & Syar, N. I. (2024). The 21st Century Education : A Systematic Literature Review Of Transforming Learning Methods To Foster Critical Thinking Skills Through Augmented Reality. *Jurnal Eduscience (JES)*, *11*(3), 601–622.

Kusumawati, I., Firdaus, F., & Oktari, V. (2024). Implementation of Meaningful Instruction Design Model Assisted by Comic on Students Understanding of Multiplication. *Jurnal Matematika Kreatif-Inovatif*, *15*(1), 97–108. https://journal.unnes.ac.id/nju/index.php/kreano.

Lin, F. L., Yang, K. L., & Chen, C. Y. (2004). The features and relationships of reasoning, proving and understanding proof in number patterns. *International Journal of Science and Mathematics Education*, *2*(2), 227–256. https://doi.org/10.1007/s10763-004-3413-z

Manurung, A. S., & Pappachan, P. (2025). The role of discovery learning in efforts to develop students' critical thinking abilities. *Journal of Education and Learning*, *19*(1), 46–53. https://doi.org/10.11591/edulearn.v19i1.21788

Methkal, Y. (2022). Role, need and benefits of mathematics in the development of society. *Journal for the Mathematics Education and Teaching Practices*, *3*(1), 23–29.

Mutianingsih, N., Hadi, S., Prayitno, L. L., Sugandi, E., & Maftuh, M. S. (2025). Eksplorasi Konstruksi Bukti Matematis Mahasiswa Menyelesaikan Soal Graf Euler : Perspektif Toulmin. *Jurnal Wahana Pendidikan, 12*(1), 41–52.

Nugraha, T. H., & Pujiastuti, H. (2019). Analisis Kemampuan Komunikasi Matematis Siswa Berdasarkan Perbedaan Gender. *Edumatica : Jurnal Pendidikan Matematika*, *9*(1), 1–7. https://doi.org/10.22437/edumatica.v9i1.5880

Prabandari, R. S., Nurhasanah, F., & Siswanto, S. (2024). Analyzing Student Creative Thinking with Wallas Theory. *International Journal of Mathematics and Mathematics Education*, *2*(2), 114–127. https://doi.org/10.56855/ijmme.v2i2.1056

Rohid, N., Suryaman, S., & Rusmawati, R. D. (2019). Students' Mathematical Communication Skills (MCS) in Solving Mathematics Problems: A Case in Indonesian Context. *Anatolian Journal of Education*, *4*(2), 19–30. https://doi.org/10.29333/aje.2019.423a

Selden, A. (2003). Validations of Proofs Considered as Texts : Can Undergraduates Tell Whether an Argument Proves a Theorem ? *Journal for Research in Mathematics Education*, *34*(1), 4–36.

Siagian, S. S., Mujib, A., & Zahari, C. L. (2022). Analisis Tingkat Kecemasan Matematika dalam Pembentukan Konsep Image Siswa. *Paradikma: Jurnal Pendidikan Matematika*, *15*(1), 8–13. https://doi.org/10.24114/paradikma.v15i1.34569

Sianturi, T. Y. (2021). *Kemampuan Representasi Dan Pemecahan Masalah Matematis Siswa Ditinjau Dari Tingkat Kecemasan Matematika*. Universitas Pendidikan Indonesia.

Slamet Kusumawardana, A., & Diantarini, M. (2021). Analissi Interpretasi Matematis dalam Mini Riset Mahasiswa Pembelajaran Berbasis Riset. *JINoP (Jurnal Inovasi Pembelajaran)*, *7*(1), 102–114. https://doi.org/10.55380/tasyri.v1i02.48

Stylianides, G. J., Stylianides, A. J., & Moutsios, A. (2024). Proof and proving in school and university mathematics education research : a systematic review. *ZDM – Mathematics Education*, *56*(1), 47–59. https://doi.org/10.1007/s11858-023-01518-y

Stylianou, D. A., Blanton, M. L., & Rotou, O. (2015). Undergraduate Students ' Understanding of Proof :

Relationships Between Proof Conceptions , Beliefs , and Classroom Experiences with Learning Proof. *International Journal of Research in Undergraduate Mathematics Education*, *1*(1), 91–134. https://doi.org/10.1007/s40753-015-0003-0

Susanto, P. C., Yuntina, L., Saribanon, E., & Soehaditama, J. P. (2024). Qualitative Method Concepts : Literature Review , Focus Group Discussion , Ethnography and Grounded Theory. *Siber Journal of Advanced Multidisiplinary (SJAM)*, *2*(2), 262–275.

Suwanto, F. R., Tobondo, Y. V., & Riskiningtyas, L. (2017). Kemampuan Abstraksi dalam Pemodelan Matematika. *Seminar Matematika Dan Pendidikan Matematika UNY*, 301–306.

Tsai, C. (2001). The interpretation construction design model for teaching science and its applications to Internet-based instruction in Taiwan. *International Journal of Educational Development*, *21*(5), 401–415.

Ye, H., Liang, B., Ng, O. L., & Chai, C. S. (2023). Integration of computational thinking in K - 12 mathematics education : a systematic review on CT - based mathematics instruction and student learning. *International Journal of STEM Education*, *10*(3), 1–26. https://doi.org/10.1186/s40594-023-00396-w

Zulkarnaen, R. (2018a). Implementasi Interpretation-Construction Design Model Terhadap Kemampuan Pemodelan Matematis Siswa SMA. *Prosiding Konferensi Nasional Penelitian Matematika Dan Pembelajarannya (KNPMP)*, 24–32.

Zulkarnaen, R. (2020). Konsepsi Siswa dalam Proses Pemodelan Matematis. *SJME (Supremum Journal of Mathematics Education)*, *4*(2), 178–187. https://doi.org/10.35706/sjme.v4i2.3638

Zulkarnaen, R. (2018b). Implementasi Interpretation-Construction Design Model Terhadap Kemampuan Siswa. *KNPMP III*, 24–32.