JURNAL EDUSCIENCE (JES)
p-ISSN : 2303 - 355X | e-ISSN : 2685 - 2217
PUBLISHED BY : LPPM of UNIVERSITAS LABUHANBATU

Inner Conflict: Mathematical Truth and Students' Perceptions of Truth

Fauzi Mulyatna[1], Dhoriva Urwatul Wutsqa[2], Syukrul Hamdi[3]

[1]Department of Mathematics Education, Universitas Indraprasta PGRI, Indonesia
[1, 2, 3]Department of Doctoral Mathematics Education, State University of Yogyakarta, Indonesia
*Email: fauzi0054fmipa.2025@student.uny.ac.id

ARTICLE INFO

Keywords:
Inner Conflict;
Mathematical Correctness;
Perception;

ABSTRACT

Purpose – This study aims to identify the forms of inner conflict potentially experienced by students, as reflected in Teaching Modules and Lesson Plans, and to propose preventive alternatives.

Methodology – This study employed a qualitative research approach with a phenomenological orientation through documentation studies of Teaching Modules and Lesson Plans developed by pre-service and in-service Professional Teacher Education students from 13 cities across Indonesia. The participants in this study consisted of documents produced by 63 students enrolled in the Professional Teacher Education (*PPG*) program. Data analysis employed triangulation techniques, including time triangulation through document analysis covering the period from 2023 to 2025, and source triangulation using Teaching Modules and Lesson Plans produced by both in-service Professional Teacher Education (*PPG*) participants and pre-service teacher candidates, previously known as *PPG Daljab* and *PPG Prajabatan*.

Findings – Beyond misconceptions, a broader range of issues can generate inner conflict among students. The findings demonstrate that mathematical conceptual contexts continue to produce conflicting interpretations of truth. Inner conflicts within students may arise from several factors: the misperception of truth caused by inaccurate lexical choices in interpreting mathematical concepts, discrepancies between mathematical results and real-world occurrences, the inappropriate selection of instructional media to explain concepts, inaccuracies in mathematical modeling, and logical fallacies in establishing connections between concrete examples and mathematical contexts.

Contribution – This study emphasizes the importance of presenting mathematical truth clearly, unambiguously, and in ways that align with students' everyday experiences. Furthermore, teachers should avoid generalizing a single mathematical truth across all contexts.

Received 10 May 2026; Received in revised form 18 May 2026; Accepted 16 July 2026
Jurnal Eduscience (JES) Volume 13 No. 4 (2026)
Available online 30 August 2026
©2025 The Author(s). Published by LPPM Universitas Labuhanbatu. This is an open-access article under the **Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International License (CC BY - NC - SA 4.0)**

INTRODUCTION

Fundamentally, mathematics deals with abstract objects of study (Carter, 2024). This characteristic is one of the factors that make mathematics difficult, as mathematical concepts are not always directly relatable to, or encountered in, everyday life. Nevertheless, this paper emphasizes mathematics learning that connects mathematical concepts with daily life through contextual learning approaches. Since mathematics involves abstract objects as well as procedural and conditional knowledge, mastering mathematical understanding requires conscious, intentional, and systematic effort. Likewise, contextual learning requires conscious engagement, particularly because it involves students' working memory.

Under certain conditions, working memory interacts with long-term memory due to the hierarchical nature of mathematical concepts. Mathematical concepts are interconnected, with some concepts serving as foundational prerequisites for others. Such conditions highlight the crucial role of prior knowledge in the learning of mathematics. (Hirsh-Pasek et al., 2015; Weisberg et al., 2016).

Prior knowledge constitutes one of the key factors in directing students' attention. (Storck et al., 2026). Students' focus on ongoing learning activities is closely associated with their prior knowledge, as it enables them to follow the learning process by connecting new information with their existing understanding. Without sufficient prior knowledge, students may fail to comprehend the intended material or questions introduced at the beginning of instruction and instead construct entirely different interpretations or associations.

Furthermore, prior knowledge also shapes perception. Incorrect perceptions may lead to conceptual misunderstandings or misconceptions. (Hendriyanto, 2026). In relation to the material being learned, prior knowledge functions as a bridge for students in acquiring new knowledge, influencing how they process information by connecting it with existing memories, interpreting it through sensory and cognitive mechanisms, and determining whether the information will be stored in short-term memory, transferred into long-term memory, or eventually forgotten. Therefore, within the learning process, teachers must appropriately position prerequisite material alongside students' prior knowledge.

Problems arise when students' prior knowledge conflicts with conditions or experiences they have previously encountered. Even differences in problem-solving methods, let alone direct contradictions, may generate negative perceptions within students. (Bruning et al., 1999). For example, students who have previously learned problem-solving techniques through private tutoring may solve limits using derivatives. Such situations can create inner conflict, as students' existing knowledge must be readjusted to align with the hierarchical structure of formal mathematics instruction. (Daheim et al., 2024). Ideally, given the hierarchical nature of mathematics, the concept of limits should be taught before differentiation or derivatives, rather than using derivatives to solve limit problems.

From the perspective of mathematics didactics, this inner conflict may also be examined through the didactical triangle, which consists of three components: the teacher, the material, and the students themselves. Within the shift toward applied mathematical contexts, the emergence of a "black box" phenomenon becomes highly possible. (Straesser, 2007). In such situations, mathematical concepts become obscured because the primary focus is merely obtaining the final numerical result. This condition becomes problematic because students are unable to understand problems conceptually and instead focus solely on the final answer. One example that once became widely discussed concerns multiplication. When multiplication is interpreted as repeated addition, the result appears identical despite differences in the repeated addends. For instance, 3×4 yields 12 both from $3 + 3 + 3 + 3$ and $4 + 4 + 4$. In this context, the concept of repeated addition becomes a "black box" due to confusion arising from the commutative property of multiplication (Straesser, 2007).

The black box is a metaphor for students' internal thinking processes that teachers or researchers cannot directly observe. (Yenil et al., 2023). The teacher can only observe the input given to the students and the outputs produced, while the mental processes that connect the two take place in a "black box". So, teachers cannot immediately know whether students understand mathematical concepts or whether students are just memorizing steps without understanding the reasoning. (Hortelano & Prudente, 2024).

From a didactic perspective, the learning process that consistently begins by informing students of learning objectives may also create problems. (Strømskag, 2024). Superficially, such objectives often appear closely related to assessment outcomes, leading to criteria that are either met or not met. However, in mathematics, there are situations in which forms and results may appear identical, while the underlying concepts students understand are fundamentally different. (Mulyatna et al., 2020). This condition may consequently lead to discrepancies within the didactical contract. (do Carmo et al., 2020).

The author recalls a personal experience related to inner conflict during third grade in elementary school. In one mathematics activity on the representation of clock hands, the author was instructed to draw a clock showing 09:00 and produced the drawing illustrated in Figure 1.

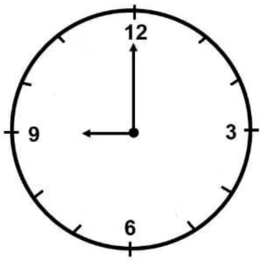

Figure 1. Clock Showing 09:00

When presenting the drawing on the blackboard, the result shown in Figure 1 was considered incorrect. However, the mistake was not immediately explained. Instead, the author was first scolded and questioned harshly about where the error lay. From a behaviorist perspective, this may now be interpreted as a form of punishment. Eventually, the author was informed that the mistake was not writing the clock numbers in full. (Edmondson, 2023). This experience generated an inner conflict that remains memorable to this day. Conceptually, the representation of the clock hands indicating the correct time was accurate. Nevertheless, it was judged incorrect solely because the clock illustration did not contain complete numerical markings. In fact, had the teacher provided an opportunity for discussion, the author could have explained that the clock at home did not display all the numbers. Within the context of contemporary educational perspectives, the author's response at that time may be associated with contextual learning, in which students present representations that are genuinely connected to their everyday experiences.

The author's experience occurred in 1997, yet similar instructional materials still exist in 2026 that appear conceptually correct from a mathematical perspective but are inconsistent with everyday practice. Conversely, everyday habits and concrete objects are often used as instructional media despite being inaccurate representations of the intended mathematical concepts. Although seemingly simple, these recurring phenomena warrant serious attention in the context of school mathematics learning. This issue becomes even more significant considering the persistent problem at the elementary education level, where mathematics is frequently taught by teachers whose educational backgrounds are not in mathematics or mathematics education.

The expansion of this problem may be directed toward the fundamental nature of mathematical truth that conflicts with students' personal perceptions of truth. Consequently, the following question emerges: Are misconceptions the only factor responsible for obscuring the essence of mathematical conceptual truth? Previous research has reviewed many related misconceptions, wrong concepts, or misunderstandings. However, it has not emphasized that the root of the problem may lie in the material presented in the learning itself. Therefore, it is important to review the material from the perspective of presenting it to facilitate the assimilation and accommodation processes, as described in cognitive conflict theory (Larsen & Misfeldt, 2021; Limón, 2001). Many studies raise questions about Realistic Mathematics Education (RME), yet many teachers still associate students' daily problems only with the context of mathematical concepts. This is also the basis for the need for this research, to reveal which material perspectives can give rise to inner conflicts in students. That is, the process of assimilation, incorporating new information into the existing knowledge structure, with the process of accommodation, changing or forming a new knowledge structure when new information cannot be explained by previous knowledge, gives rise to conflict. Assimilation with these conflicting accommodations can lead to inner conflicts. Students feel right based on assimilation with daily knowledge/experience, but not in accordance with the accommodation process in the formation of mathematical concepts. The conceptual change is disrupted.

In fact, mathematical knowledge is built through interaction between students, teachers, and the learning environment (milieu), which is deliberately designed to encourage students to discover mathematical concepts independently. (Hortelano & Prudente, 2024). Given that students' daily experiences do not align with the truth of mathematical concepts, the environmental component in didactical situations theory becomes unfulfilled. So, from the components of students, teachers, and the environment, it is clear that we cannot control the learning environment of students. Through this research, what can be prevented from the perspective of teachers, especially in packaging learning materials, can be identified.

The researcher's subjective phenomenological experiences, therefore, become important to reflect upon within a broader scope. In this study, these experiences are further reflected through findings obtained from documentation studies of Lesson Plans and Teaching Modules. By raising these seemingly simple issues, the study is expected to generate broader educational implications, at least by encouraging teachers to pay closer attention to the materials they intend to teach, beginning from the earliest stage of instruction, namely, instructional planning. In the future, teachers should carefully consider the abstract nature of mathematical objects when contextualizing them through concrete examples. Most importantly, teachers should avoid becoming trapped in the mindset that mathematics always has a single numerical answer or that every mathematical problem necessarily has a single definitive solution.

METHODOLOGY

Research Design

The methodological approach employed in this study is qualitative research using a phenomenological perspective. The emphasis of phenomenological analysis lies in individuals' subjective experiences, which strengthen this study as it is grounded in the researcher's personal experiences. (Mertens, 2019). These personal experiences have been continuously perceived from the researcher's years in elementary school through becoming a lecturer in mathematics education. The expansion of these personal experiences is subsequently reflected in Teaching Modules and Lesson Plans, which frequently contain conceptual contexts that are inconsistent with students' everyday experiences. Such interpretations may be further explored through phenomenological studies designed to describe and interpret experiences by identifying the meanings of experiences encountered by individuals engaged in the same field. (Ary et al., 2019)

Furthermore, phenomenology is defined as an investigative design derived from philosophy and psychology in which researchers describe individuals' lived experiences regarding a particular phenomenon, as reflected through the documentation analysis of Teaching Modules and Lesson Plans. (Creswell & Creswell, 2017). This description ultimately aims to identify the essence of similar experiences shared by others who have encountered the phenomenon.

Participant

This study originated from phenomenological reflections identified within Teaching Modules and Lesson Plans. Therefore, the participants in this study consisted of documents produced by 63 students enrolled in the Professional Teacher Education (*PPG*) program, including both pre-service teacher candidates and in-service teachers. These participants were drawn from various cities across Indonesia, including Jakarta, Tangerang, Banyuwangi, Boyolali, Tasikmalaya, Pandeglang, Bekasi, Ciamis, Magelang Regency, Sidoarjo, Kediri, Bangkalan, and Karawang. The selected participants strengthen the observed phenomenon, considering that they had at least completed undergraduate studies in mathematics or mathematics education.

Furthermore, participants in the in-service Professional Teacher Education program already possessed professional teaching experience.

Instrument

The primary data collection instruments in this study were Teaching Modules and Lesson Plans. These instructional documents constitute mandatory components within the learning process. Furthermore, the Teaching Modules and Lesson Plans had been validated by at least the lecturers or examiners responsible for assessing Professional Teacher Education (PPG) performance, as the selected participants were individuals who had completed teacher certification and obtained professional teaching certificates.

Data Collection

The focus of this study is the inner conflict arising from mathematical conceptual contexts as reflected in document analysis. Accordingly, the data sources consisted of Teaching Modules and Lesson Plans collected from participants in the Professional Teacher Education (*PPG*) program, including both in-service teachers and pre-service teacher candidates, previously referred to as *PPG Daljab* and *PPG Prajabatan*. The data were obtained from the implementation of the *PPG* program during the period 2023–2025.

Data Analysis

The qualitative data analysis in this study employed the interactive analysis model proposed by Miles et al. (2014), consisting of the stages of data collection, data reduction, data presentation, and conclusion drawing or data verification. Following data collection, the data were analyzed through a reduction process. Activities at the data reduction stage involved reviewing instructional materials and assessments to identify whether they contained the phenomenon under investigation. These mathematical truths are, namely, inconsistent with students' everyday experiential knowledge. Data review and categorization were conducted using repeated checks to minimize the risk of classification errors.

Subsequently, the researcher presented the data by defining and organizing them according to their classifications, while considering the study's focus and objectives. The final stage of the analysis involved concluding a definitive interpretation of the research findings. Referring to the participants involved in this study, the triangulation techniques employed included time triangulation, through document analysis covering the period from 2023 to 2025, and source triangulation, using Teaching Modules and Lesson Plans produced by both in-service Professional Teacher Education (*PPG*) participants and pre-service teacher candidates, previously known as *PPG Daljab* and *PPG Prajabatan*. Although the documents shared similar formats, they possessed different characteristics. Individuals with professional teaching experience designed documents originating from in-service PPG participants, whereas those produced by pre-service *PPG* participants were developed by individuals who had not yet formally assumed the role of teachers.

FINDINGS

The findings of this research narrate the presentation of materials represented in the Teaching Modules/RPP, which may lead to ambiguity. The ambiguities identified in this documentary study extend beyond what is commonly defined as misconceptions. This study reveals that throughout three years (2023–2025), such ambiguities consistently persist and are distributed across 10 cities in Indonesia. Consequently, reflecting on the author's personal experience, these subjective observations constitute a phenomenological finding, indicating that this phenomenon has persisted from 1997 through 2025 and is reflected across ten cities in Indonesia.

Material Findings Capable of Inducing Conflict

The collected Teaching Modules and Lesson Plans (*RPP*) were analyzed and categorized into those that contain no contradictions and those that may conflict with students' perceptions. The initial context identified pertains to Algebraic Materials, specifically the topic of Addition and Subtraction of Algebraic Expressions, particularly in the construction of the concept of variables. An example of this data reduction is illustrated in Figure 2.

Figure 2. A Maze Accessible through Three Distinct Paths

Based on Figure 2, the subject presents a problem as follows:

> Look, there are three paths over there; let us assist Father and Andi. By calculating the quantity of fruit along each path, Father and Andi wish to determine the total amount of fruit using algebraic expressions.

$$\begin{array}{l} Jalur\ 1 = \cdots x + \cdots y + \cdots z \\ Jalur\ 2 = \cdots x + \cdots y + \cdots z \\ \underline{Jalur\ 3 = \cdots x + \cdots y + \cdots z} + \\ \qquad\quad\ = \cdots x + \cdots y + \cdots z \end{array}$$

The problem context presented by the subject may induce inner conflict in the future. This conflict arises because it shapes a certain perception within the student if left unaddressed or without proper emphasis from the teacher. The variables appearing in the problem can lead to an interpretation that points toward counting units. The variables x, y, and z, which respectively represent the images of bananas, mangoes, and oranges, can be interpreted as x = bananas, y = mangoes, and z = oranges. This is evident from the fill-in-the-blank format provided for the answers. A similar context was also identified in three other Teaching Modules.

There is a critical need to clarify variable definitions to ensure that practices that appear correct through habit and have become commonplace align with formal mathematical principles. In daily life, students frequently encounter sentences such as person x enjoys helping their parents, where x functions as a variable representing a specific individual's name or identity. However, using x to represent 'banana' is mathematically imprecise if the intended reference is to the quantity of bananas.

This imprecision occurs when the result is known to be 11 bananas; if the initial definition is x = banana, it should logically result in the notation 11x. It would be mathematically accurate if x were defined as the number of bananas. This consequence would result in the 11 bananas being represented as 11x bananas. This inner conflict arises because, mathematically, the process may still yield the correct numerical solution. However, the final answer can lead to subsequent confusion for the student. From the definition x = banana, the researcher must reiterate that 11 bananas would, by that definition, become 11x. This paper is crucial for mathematics teachers to recognize that a student's perception that the final result has not yet been fully resolved may actually be valid. This is the inner conflict stemming from an imprecise definition of variables. The situation differs entirely if x is defined as representing the quantity of bananas.

Still within the domain of algebra as applied to Systems of Linear Equations in Three Variables, documentary evidence reveals a phenomenon regarding the use of distinct symbols for variables, coefficients, and constants. In one instance, an equation was written as a + 3b – 2c = 7, where a, b, and c were subsequently identified as the variables. It must be carefully noted that, in mathematics, a is not equivalent to a, if they are typographically distinct symbolically denoted as $a \neq$a. Although they share the same letter, different fonts or styles in mathematics represent different conceptual contexts. Mathematics is a symbolic language in which every notation represents a specific concept; even the distinction between uppercase and lowercase letters can carry different meanings. In this context, it must be strictly maintained that $a \neq$a. If $a =$a, it may induce a conflicting perception of truth within the student. For example, if it is established that $a = 5$, then as a consequence of the premise $a =$a, a student might interpret the word "adalah" (meaning 'is') as "5d5l5h", because every a is replaced by the value 5. This identical context was identified in four different Teaching Module/RPP documents."

In the material on Arithmetic Operations on Whole Numbers up to 100,000, specifically under the topic of Multiplication, it was also found that a single answer form does not always represent mathematical truth. A subsequent issue identified pertains to the currency values used in payment transactions. This problem necessitates specific constraints regarding the use of 'exact change,' or requires teachers to provide multiple valid solutions. This is particularly relevant when the instructional command is, 'Circle the amount of money to be used for payment!' The context found within the Teaching Modules is represented in

Figure 3.

Figure 3. Currency Utilized for Payment Transactions

The delivery of contextual materials related to currency values, as illustrated in Figure 3, requires meticulous precision. Inner conflict may arise when formal mathematical correctness is confronted with the realities of daily experience. For instance, a student possessing two banknotes, one of Rp20,000.00 and another of Rp10,000.00, who purchases an item priced at Rp16,000.00, would typically receive Rp4,000.00 in change. This occurs because, according to common practice and social norms, only the Rp20,000.00 banknote would be tendered. In contrast, within a strictly mathematical context, if an individual possesses a total value of Rp30,000.00 and purchases an item for Rp16,000.00, the calculated change would be Rp14,000.00.

Currency is also frequently modeled as a circle, a phenomenon observed in the Lesson Plans (RPP) presented by the subjects. However, within the context of Euclidean Postulates, a circle is defined as the set of all points equidistant from a single central point. Consequently, the mathematically accurate interpretation of a circle refers only to its boundary (circumference). In a concrete sense, more appropriate examples would include bicycle wheels, rings, or bracelets.

The next material, which contains potential contradictions, pertains to vectors. Vectors are often illustrated using GPS, which leads students to associate the concept with Google Maps or ride-hailing services immediately. A problem arises if it is not emphasized that the path from the starting point to the destination typically does not form a straight line segment. Conversely, the vector concept being discussed defines the relationship between two points as a straight directed line segment (ray). For students who have studied physics, equating a GPS route with a straight line segment may also create an inner conflict, as it risks obscuring the fundamental distinction between displacement and distance.

Still within the topic of vectors, the use of examples such as drones, pillars, or pipes must not be limited to merely stating that these objects are applications of vector concepts. Without proper emphasis on which specific part of the object embodies the vector concept, inner conflict is likely to recur. From the perspective of biologically primary knowledge, a student perceives a pipe as an inanimate, straight, and generally hollow cylindrical object. (Geary, 2008). Conversely, biologically secondary knowledge dictates that a vector must possess direction; thus, the student may struggle to associate the physical properties of a pipe with the directional requirement of a vector.

Instructional roles expect teachers to provide explicit emphasis when teaching the concept of fractions. In the Teaching Modules analyzed, fractions are often derived from the root word 'break' (in Indonesian, *pecah*). This requires careful clarification. While 'breaking' can illustrate a part-to-whole relationship, teachers must provide illustrative examples of the terms 'break' and 'fraction' as used in non-mathematical contexts.

For instance, in financial transactions, the term 'denominations' (in Indonesian, pecahan) is frequently used, such as '50,000-unit denominations,' yet this context does not refer to the mathematical concept of fractional numbers. Similarly, the request to 'break' (exchange) a 50,000 banknote into five 10,000 banknotes does not align with the mathematical definition of a fraction. In a strictly mathematical context, 'breaking' a 50,000 banknote would imply physically dividing the paper into five equal parts, each representing one-fifth of the original value.

Still, regarding the topic of fractions, students are introduced to the concept of equivalent fractions. (Jarrah et al., 2022). Once students have mastered this concept, teachers should avoid being constrained by the expectation of a single, identical final numerical form. Unless the problem specifically requires simplifying fractions to their lowest terms, teachers should allow for the mathematical validity of both. $\frac{3}{7}$ and $\frac{9}{21}$. If a student's answer is marked incorrect simply because it was not simplified, it may induce inner conflict. The concept of equivalent fractions, which they have previously learned, would then appear to have no consequence on the validity of their answers.

Another context identified within the modules concerns decimal numbers, specifically issues surrounding rounding. Teachers need to emphasize the distinction between rounding up and rounding down. Furthermore, teachers must provide logically sound justifications; solutions should not merely adhere to mathematical procedures if they lack practical logic. For instance, when calculations involve human beings and result in non-integers, the rounding context should logically necessitate truncating the decimal. The decimal portion should be disregarded as it would otherwise induce inner conflict; if a result is 5.974, should it become 6? This leads to the further question: Does 0.974 imply an incomplete person? Similarly, when purchasing fuel such as Pertalite, amounts of Rp15,125 or Rp15,457 are commonly rounded to Rp15,500 or even Rp16,000 in practice. Teachers must possess broad contextual knowledge to illustrate how authentic mathematics operates within these everyday scenarios.

As previously discussed, the improper use of symbols can lead to varying perceptions of truth, which, from the student's perspective, cannot be deemed incorrect. Furthermore, as shown in Figure 4, imprecise angle depictions can lead to conflicting perceptions, particularly in related mathematical content. As explained in the introduction, prior knowledge significantly influences how students perceive the validity of a given problem. In this case, the task is to determine the angle measurements shown in Figure 4. However, given their prior knowledge, students cannot be faulted for concluding that none of the illustrations in Figure 4 have a measurable value. This occurs because students have prior knowledge of the distinctions among lines, line segments, and rays. These concepts are often overlooked and erroneously treated as identical. Consequently, if the Teaching Module defines an angle as a conceptual construct formed by at least two intersecting rays sharing a common endpoint (vertex), then the illustrations in Figure 4 do not qualify as angles, as they are composed of line segments rather than rays. Educators must exercise extreme caution when presenting these conceptual contexts.

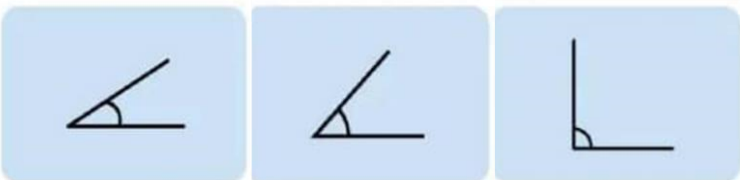

Figure 4. Inaccurate Depiction of Angles

The next topic capable of inducing a perception of truth that contradicts mathematical concepts pertains to data representation. There is a shift in meaning where two distinct ideas are treated as identical, which is conceptually erroneous in mathematics. The teacher's presentation seems to equate the meaning of 'average' (mean) with equal distribution. Consider the following problem: “SD Negeri 1 Kagungan has six classes, from Grade 1 to Grade 6. If each class has an average cash fund of Rp16,000, determine the total cash fund at *SD*

Negeri 1 Kagungan! This creates an inner conflict within the student regarding the perception of average versus an equal amount. This occurs because the solution provided in the module is 6 x Rp16,000. Consequently, the average of Rp16,000 per class is interpreted as if each class has an identical cash fund of exactly Rp16,000.

The following problem also appears to be correctly resolved within a mathematical conceptual context. The contextual problem provided is: "A road is 5 km long. If streetlights are to be installed every 100 meters, how many streetlights are required?" In the context of integer arithmetic, the solution provided relies on division. Thus, 5 km is converted to 5,000 m; for every 100 m, a lamp is installed, resulting in 5,000:100 = 50 lamps. This is the solution presented by the teacher in the Teaching Module. However, this is precisely where an inner conflict arises. A student with a different perspective, grounded in real-world experience, will arrive at a different answer. It is important to remember that, in reality, installing streetlights always begins at the starting point and concludes at the end point; therefore, the total number of lamps should be 51. This explanation is vital for teachers as a form of reflection and emphasis: mathematical concepts in daily life are influenced by assumptions and external factors. Consequently, when engaged in mathematical modeling, students should be encouraged to consider various possibilities as assumptions emerge.

In 2015, the author conducted research on misconceptions, and by 2025, the same contexts were still being encountered. This persistence reinforces the designation of this research method as phenomenology, as the phenomena continue to recur and are still observed even after more than a decade.

The phenomenology of these misconceptions is evident in the subject of integers, specifically in integer arithmetic operations (addition and subtraction). The problems presented,

The result of $-8+5-8\,+\,5-8+5$ is ...

The explanation presented, $-8\,+\,5\,=\,-3\,-\,8\,+\,5\,=\,-3\,-\,8\,+\,5\,=\,-3$

Based on prior research, this situation may induce inner conflict. From the perspective of the final result, the correct numerical value is obtained. However, the conceptual meaning of the equals sign (=) which can be interpreted as a result that can then be subjected to further arithmetic operations, is precisely what renders this process incorrect. The meaning of the equals sign (=) dictates that each part separated by the symbol must possess an identical value. Consider that,

$$-8\,+\,5\,\neq\,-3\,-\,8\,+\,5\,-3\,-\,8\,+\,5\neq\,-3\,-\,8\,+\,5\,-3\,-\,8\,+\,5\,\neq\,-3$$

Furthermore, the aforementioned findings are categorized. The objective of this categorization is to alert the readers of this research, ensuring that they avoid repeating these errors or presenting them in different contexts with similar forms. (Kurudirek et al., 2025).

Categories of Identified Inner Conflicts

Inaccuracy in Lexical Equivalents for Interpreting Mathematical Concepts.

Mathematical subject matter occasionally employs lexical equivalents commonly used in daily life. Oftentimes, this is precisely what induces inner conflict within students. Students possess a perspective of truth derived from everyday contexts, which, naturally, is not entirely incorrect. This research presents data on the use of the word "break" (pecah) in the context of fractions, and clarifies that the meaning of "average" (mean) is not always synonymous with "equal distribution".

Discrepancies Between Mathematical Results and Real-World Occurrences

Results derived from mathematical calculations based on abstract objects of study may yield divergent outcomes under certain conditions. Reality does not always present ideal circumstances, and there are often extraneous factors to consider. In the context of this research, the calculation regarding the required number of streetlights demonstrates that observations of actual conditions must accompany mathematical computation. There is a need for explicit emphasis and the establishment of assumptions regarding the process being undertaken. If mathematical calculation is regarded as the sole source of truth, it may induce inner conflict within students. Similarly, in the contextual examples of currency usage, daily experiences involving monetary value must be taken into account; the representation of currency value is not always identical to the value of pure numbers.

Inappropriateness in Selecting Instructional Media to Explain Mathematical Concepts

Due to the abstract nature of mathematical objects, the use of concrete objects to represent mathematical contexts requires a precise process of abstraction. A common error identified in this research, and one that frequently occurs, is the illustration of a circle. A coin often exemplifies a circle. This can induce inner conflict within students. While circles drawn in Cartesian coordinates or defined by equations of circles represent only the boundary (the locus of points), the example provided often encompasses the entire disk, including the interior. More accurate examples include rings, bracelets, wheels, and hula hoops.

Inaccuracy in Mathematical Modeling

Making assumptions is an essential requirement in mathematics if one aims to achieve accurate and applicable results. This research identifies specific cases within the rounding process. When the result of a calculation is a living being, such as a human, the appropriate approach is to round down (truncate). Conversely, when purchasing fuel, the common practice is to round up. Furthermore, when predicting future events, it is necessary to assume that no extraneous factors will influence the outcome, or those factors must be integrated into the calculation through explicit assumptions.

Logical Fallacies in Establishing Connections Between Concrete Examples and Mathematical Concepts

Drawing parallels between concrete objects or daily activities and mathematical concepts requires careful consideration of their logical coherence and relevance. Data in this research indicates that illustrating the concept of vectors by simply referencing GPS or Google Maps can be ineffective if it lacks specific emphasis on where the vector component actually resides. Similarly, when a pipe is used as an example, it must be accompanied by a clear description of the specific scenario or physical property of the pipe that aligns with the intended mathematical concept.

DISCUSSION

Research findings indicate that inner conflicts arise from the intersection of formal learning experiences and daily life processes. Students are perpetually engaged in mathematical activities, whether consciously or unconsciously. Given these findings, it is essential to provide alternative preventive solutions. Since these conflicting perspectives emerge from two sides, everyday experience and mathematical conceptual contexts, the alternative for emphasizing correct concepts must be implemented through instructional design and the teacher's perspective. It is impossible to control all of a student's daily activities outside of school so that they strictly align with formal mathematical contexts. Therefore, the role of the teacher is paramount; educators must continually enhance their professional competence, particularly in subject-matter expertise. In doing so, instructional delivery that triggers inner conflict can be avoided. This research intentionally provides extensive narrative reviews of mathematical conceptual contexts to highlight where these conflicts arise. By presenting these alternative perspectives of truth, it is hoped that teachers will exercise greater caution and refinement in their pedagogical delivery.

Discussion of Instructional Design

Learning is an active process in which students construct mathematical understanding through meaningful experiences (Annajmi et al., 2026). This principle is embedded in the implementation of Problem-Based Learning (PBL) and other pedagogical approaches, such as Contextual Teaching and Learning (CTL), Realistic Mathematics Education (RME), which has been adapted in Indonesia as *Pendidikan Matematika Realistik Indonesia* (*PMRI*), or the assignment of contextual projects through Project-Based Learning (PjBL). These methods involve exploring contextual problems (e.g., fractions, monetary values, applications of algebraic forms, and GPS). In the context of GPS, specific emphasis must be placed on its relationship with vector concepts; otherwise, it may trigger inner conflict within students. From the perspective of evolutionary educational psychology, students possess biologically primary knowledge, intuitive knowledge gained from daily life, such as using Google Maps or ride-hailing applications. In these experiences, they automatically associate displacement with travel paths, which are rarely straight lines. Conversely, biologically secondary knowledge acquired through formal instruction associates vectors with displacement, typically represented as straight lines. Effective learning, therefore, does not merely involve memorizing the definition of collinear points; it requires understanding the core concept of vectors, analyzing the collinearity of points through component ratios, and reflecting on common computational errors.

Instruction refers to a series of planned activities designed by the teacher to facilitate learning experiences. (Avishai et al., 2025). Within the framework of Problem-Based Learning (PBL), this encompasses stages such as problem orientation, grouping based on diagnostic assessment results, the utilization of student worksheets (*LKPD*), and TPACK-based media, culminating in presentation and reflection. Furthermore, instructional design is reflected in the systematic planning of the learning process, which includes learning outcomes, learning objectives (following the ABCD criteria), the PBL model, and the Teaching at the Right Level (TaRL) approach. This design provides a step-by-step scenario spanning the introduction, core activities, and closing, along with the strategic planning for diagnostic, formative, and summative assessments.

Instructional design that incorporates learning objectives (adhering to the ABCD elements) serves as a fundamental guide within the learning contract. From a didactic perspective, the student's learning process must align with the teacher's intended goals. This definition must be clear and concrete. The essence of a learning objective lies not merely in the assessment results, but in the conceptual context understood by the student as envisioned by the teacher; this alignment constitutes what is known as adherence to the didactic contract. (do Carmo et al., 2020; Strømskag & Chevallard, 2023).

Meanwhile, the psychology of learning is reflected in how instruction accounts for cognitive aspects (conceptual understanding of mathematics), affective domains (motivation through real-world contexts), metacognition (reflection on difficulties and learning strategies), and mathematical resilience through non-cognitive assessments. Learning is viewed as a mental and affective process involving student comprehension, motivation, and self-reflection. In this regard, the teacher must play a pivotal role in the metacognitive phase to ensure that the reflective process results in mathematically sound concepts.

Within the affective domain, the teacher's ability to establish valid links between concepts and real-world contexts is crucial. Ensuring precise alignment between the real-world scenario and the mathematical concept prevents the formation of a "black box" in which students follow procedures without understanding the underlying logic. (Straesser, 2007).

Furthermore, associating mathematical concepts with daily life requires the explicit use of assumptions. This is vital because phenomena are often influenced by extraneous factors that could otherwise distort the mathematical model.

Teacher Role

The Honest Teacher

The implementation of mathematics instruction is inseparable from the teacher's role. (Ansari et al., 2026). Traditionally, the teacher is often perceived as an omniscient figure, a primary source of truth and knowledge. However, in mathematical contexts where multiple solution paths exist, or where results are non-exact or take different forms, teachers should remain open to evaluating validity from various perspectives while maintaining mathematical conceptual correctness as the ultimate benchmark. In the current era, many students receive additional instruction through private tutoring outside of school, which frequently introduces alternative problem-solving methodologies. If a teacher is not yet able to reconcile these differing perspectives, they should honestly request time from the student to analyze and understand that alternative approach. This transparency is crucial to avoid immediate, arbitrary judgment that labels a method "incorrect" simply because it differs from the conventional approach taught in class.

The Teacher's Role in Facilitating Prerequisite and Prior Knowledge Construction

Given the hierarchical nature of mathematical study, teachers must align their perceptions with students' existing understanding. A correct didactic contract must be established from the outset to ensure that the concepts students comprehend and believe to be true are not inherently flawed. (Strømskag & Chevallard, 2023). Furthermore, teachers must commit to lifelong learning and remain open to existing research. This underscores the significance of this study, which reviews phenomena that recur annually and are often mistakenly accepted as truth, such as the data in this research identifying the side of a coin as a "circle." Teachers must continuously enhance their professional competence. Given that the documents analyzed in this study originate from professional educators, this underscores that, while human error is inevitable, striving to minimize it through continuous improvement is a fundamental obligation for every teacher.

Prioritizing Conceptual Substance over Numerical Uniformity in Problem-Solving

Inner conflict among students can also stem from discrepancies in final results that the teacher subsequently deems incorrect. (Daheim et al., 2024; Park & Ramirez, 2022). Therefore, educators need to prioritize the conceptual substance of a solution. If a problem allows for diverse final forms or representations, the teacher must reinforce that these differing answers remain valid, provided they are mathematically sound and permissible within the conceptual context.

Facilitating Discussion for Divergent Solutions

It is essential to recognize that mathematics does not always yield exact answers or uniform solution formats; therefore, discrepancies in answers must be addressed through open discussion. (Lampert, 1990; Yan et al., 2024). When a teacher is unable to immediately discern a student's line of reasoning or cognitive flow, engaging in dialogue serves as a vital pedagogical bridge. However, when a student’s answer or methodology is demonstrably incorrect, the teacher must be able to identify and explain the nature of the error precisely.

This study is of paramount importance as it brings necessary attention to prevailing narratives that may inadvertently misalign with absolute mathematical truths. The concern is that such conceptual misalignments could accumulate and solidify within a student’s biologically primary knowledge, biologically secondary knowledge, and overall prior knowledge. (Bruning et al., 1999). Given the hierarchical nature of mathematics, the subject must be constructed on prerequisite material and initial abilities that are free of conflicts, specifically those arising from discrepancies between mathematical truth and a student's pre-existing perceptions. Furthermore, this research serves as a gateway for future researchers to provide even more comprehensive categorizations across diverse contexts or to conduct in-depth analyses of various other mathematical topics.

CONCLUSION

Inner conflicts within students may arise from several factors that are the findings of the research: the misperception of truth caused by inaccurate lexical choices in interpreting mathematical concepts, discrepancies between mathematical results and real-world occurrences, the inappropriate selection of instructional media to explain concepts, inaccuracies in mathematical modeling, and logical fallacies in establishing connections between concrete examples and mathematical contexts. To minimize or avoid these issues, alternative solutions must be implemented through strategic instructional design and the teacher's proactive role. This study shows that inner conflict arises not only from student misconceptions but also from learning design, media, context, and the mathematical representations used by teachers. The implication is that in terms of Didactical Situations Theory (students, teachers, and learning environment (milieu)), they cannot control the student's life environment. However, the teacher can present the student environment in the right learning context.

ACKNOWLEDGMENT

The researcher wishes to express sincere gratitude to Prof. Wahyu Setyaningrum, S.Pd., M.Ed., Ph.D., and Prof. Dr. Ariyadi Wijaya, S.Pd.Si., M.Sc. Through limited dissemination within the Scientific Article Writing and Mathematics Didactics courses, this research benefited from valuable discussions and constructive feedback, which facilitated its completion. It is hoped that this study will provide significant benefits, particularly for the mathematics education community.

REFERENCES

Annajmi, A., Kuswandi, D., Setyosari, P., & Praherdhiono, H. (2026). Integrating multiple representations and local cultural contexts into problem-based learning within a flipped classroom: A new framework for mathematics education. *Social Sciences & Humanities Open*, *13*, 102797. https://doi.org/10.1016/j.ssaho.2026.102797

Ansari, S., Sajadi, S. M., & Assareh, A. (2026). Examining the Relationship Between Mathematics Teaching Content and Teaching Methods: A Phenomenological Study (Lower Secondary Mathematics Teachers in District 3 of Tehran). *Assessment and Practice in Educational Sciences*, 1–12.

Ary, D., Jacobs, L. C., & Razavieh, A. (2019). *Introduction to research in education, Tenth Edition.* Cengage Learning.

Avishai, T., Palatnik, A., & Ben-David Kolikant, Y. (2025). Amplifiers and filters in teacher learning of student-centered mathematics instruction. *Teaching and Teacher Education*, *156*, 104943. https://doi.org/10.1016/j.tate.2025.104943

Bruning, R. H., Schraw, G. J., & Ronning, R. R. (1999). *Cognitive psychology and instruction*. ERIC.

Carter, J. (2024). Mathematical understanding–Common themes in philosophy and mathematics education. *The Journal of Mathematical Behavior*, *76*, 101202. https://doi.org/10.1016/j.jmathb.2024.101202

Creswell, J. W., & Creswell, J. D. (2017). *Research Design: Qualitative, Quantitative, and Mixed Methods Approaches*. California: SAGE Publications.

Daheim, N., Macina, J., Kapur, M., Gurevych, I., & Sachan, M. (2024). Stepwise verification and remediation of student reasoning errors with large language model tutors. *Proceedings of the 2024 Conference on Empirical Methods in Natural Language Processing*, 8386–8411.

do Carmo, F. M. A., de O. Faustino, J. A., de Lima, M. V. M., Felício, M., Neto, H. B., & Cerqueira, G. S. (2020). The Didactic Contract from the Perspective of the Theory of Didactical Situations: An Integrative Review. *Int J Innov Educ Res.*, *8*(7), 123–134.

Edmondson, A. (2023). *Right kind of wrong: Why learning to fail can teach us to thrive*. Random House.

Geary, D. C. (2008). An evolutionarily informed education science. *Educational Psychologist*, *43*(4), 179–195. https://doi.org/10.1080/00461520802392207

Hendriyanto, A. (2026). Knowledge of sets in curricular transposition: A study of didactic transposition phenomena in Indonesia. *Social Sciences & Humanities Open*, *13*, 102686. https://doi.org/10.1016/j.ssaho.2026.102686

Hirsh-Pasek, K., Zosh, J. M., Golinkoff, R. M., Gray, J. H., Robb, M. B., & Kaufman, J. (2015). Putting Education in "Educational" Apps: Lessons From the Science of Learning. *Psychological Science in the Public Interest*, *16*(1), 3–34. https://doi.org/10.1177/1529100615569721

Hortelano, J. C., & Prudente, M. (2024). Effects of the Theory of Didactical Situations' Application in Mathematics Education: A Metasynthesis. *Journal of Pedagogical Research*, *8*(3), 246–262. https://doi.org/10.33902/JPR.202426908

Jarrah, A. M., Wardat, Y., & Gningue, S. (2022). Misconception on addition and subtraction of fractions in seventh-grade middle school students—Eurasia *Journal of Mathematics, Science and Technology Education*, *18*(6), em2115.

Kurudirek, A., Karim, B., Sarhang, D., & Tulqin, S. (2025). Math misconceptions: Mistakes, misunderstanding, and confusion. *Educenter: Jurnal Ilmiah Pendidikan*, *4*(1), 16–25.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal*, *27*(1), 29–63.

Larsen, D. M., & Misfeldt, M. (2021). Fostering mathematical reasoning in inquiry-based teaching–the role of cognitive conflicts. *NOMAD Nordic Studies in Mathematics Education*, *26*(2), 7–23. https://doi.org/10.7146/nomad.v26i2.149162

Limón, M. (2001). On the cognitive conflict as an instructional strategy for conceptual change: A critical appraisal. *Learning and Instruction*, *11*(4–5), 357–380. https://doi.org/10.1016/S0959-4752(00)00037-2

Mertens, D. M. (2019). *Research and evaluation in education and psychology: Integrating diversity with quantitative, qualitative, and mixed methods*. Sage Publications.

Miles, M. B., Huberman, A. M., & Saldana, J. (2014). *Qualitative data analysis*. Sage Publications Inc.

Mulyatna, F., Nurrahmah, A., & Seruni. (2020). Look Same but Different: Misconceptions in Linear. *Proceedings of the SEMANTIK Conference of Mathematics Education (SEMANTIK 2019)*, 1–5. https://doi.org/10.2991/assehr.k.200827.107

Park, D., & Ramirez, G. (2022). Frustration in the classroom: Causes and strategies to help teachers cope productively. *Educational Psychology Review*, *34*(4), 1955–1983.

Storck, A., Schönefeld, E., Bellstedt, M., Barbian, B., Janssen, M., Seifert, K. E., & Darici, D. (2026). Instructional Design Unlocks the Effects of Prior Knowledge Activation in Surgical Education. *Journal of Surgical Education*, *83*(5), 103915. https://doi.org/10.1016/j.jsurg.2026.103915

Straesser, R. (2007). Didactics of mathematics: more than mathematics and school! *ZDM Mathematics Education*, *39*(1), 165–171. https://doi.org/10.1007/s11858-006-0016-x

Strømskag, H. (2024). Complementary dimensions of the Theory of Didactic Situations in Mathematics and the Theory of Social Interactionism: Synthesizing the Topaze effect and the funnel pattern. *The Journal of Mathematical Behavior*, *76*, 101194. https://doi.org/10.1016/j.jmathb.2024.101194

Strømskag, H., & Chevallard, Y. (2023). Breaches of the didactic contract as a driving force behind learning and non-learning: a story of flaws and wants. *Teaching Mathematics and Its Applications: An International Journal of the IMA*, *42*(1), 52–64.

Weisberg, D. S., Hirsh-Pasek, K., Golinkoff, R. M., Kittredge, A. K., & Klahr, D. (2016). Guided Play: Principles and Practices. *Current Directions in Psychological Science*, *25*(3), 177–182. https://doi.org/10.1177/0963721416645512

Yan, V. X., Sana, F., & Carvalho, P. F. (2024). No simple solutions to Complex problems: Cognitive science principles can guide but not prescribe educational decisions. *Policy Insights from the Behavioral and Brain Sciences*, *11*(1), 59–66.

Yenil, T., Arslan, Ç., & Broutin, M. S. T. (2023). Triangle Inequality Concept Teaching: The Theory of Didactic Situations Case. *Journal of Pedagogical Research*, *7*(4), 14–29. https://doi.org/10.33902/JPR.202318961